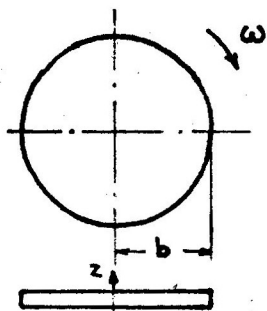




# INSTYTUT MECHANIKI I PODSTAW KONSTRUKCJI MASZYN WYDZIAŁU MECHANICZNEGO

Wytrzymałość Materiałów III - Wykład 6

## Nośność graniczna wirujących tarcz kołowych i pierścieniowych



r. r. w.

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} + \gamma \omega^2 r = 0 \quad (1)$$

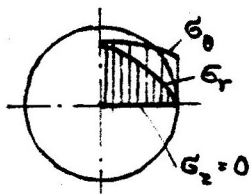
HMH

$$\sigma_r^2 + \sigma_\theta^2 - \sigma_r \sigma_\theta = \sigma_0^2 \quad (2)$$

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$$\text{TG} \quad \sigma_I - \sigma_{III} = \sigma_0 \quad (3)$$

w zakresie sprężystym :



$$\left. \begin{aligned} \sigma_I &= \sigma_\theta \\ \sigma_{II} &= \sigma_r \\ \sigma_{III} &= \sigma_z = 0 \end{aligned} \right\} \quad (4)$$

zakładamy podobnie :

$$\sigma_I = \sigma_\theta, \quad \sigma_{II} = 0.$$

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} + \gamma \omega^2 r = 0 \quad (5)$$

$$\sigma_\theta = \sigma_0 = \text{const.} \quad (6)$$

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_0}{r} + \frac{\gamma}{g} \omega^2 r = 0 \quad (7)$$

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r}{r} = \frac{\sigma_0}{r} - \frac{\gamma}{g} \omega^2 r \quad (8)$$

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r}{r} = 0 \quad (9)$$

$$\frac{d\sigma_r}{\sigma_r} = - \frac{dr}{r} \quad (10)$$

$$\ln \sigma_r = - \ln r + \ln \bar{C} \quad (11)$$

$$\sigma_r = \frac{\bar{C}}{r} \quad \bar{C} = \bar{C}(r) \quad (12)$$

$$\frac{\bar{C}'}{r} - \frac{\bar{C}}{r^2} + \frac{\bar{C}}{r^2} = \frac{\sigma_0}{r} - \frac{\gamma}{g} \omega^2 r \quad (13)$$

$$\bar{C}' = \sigma_0 - \frac{\gamma}{g} \omega^2 r^2 \quad (14)$$

$$\bar{C} = \sigma_0 r - \frac{\gamma}{g} \omega^2 \frac{r^3}{3} + C \quad (15)$$

$$(12) \rightarrow \sigma_r = \sigma_0 - \frac{\gamma}{3g} \omega^2 r^2 + \frac{C}{r} \quad (16)$$

$$\sigma_\theta = \sigma_0 \quad (17)$$

Warunki brzegowe dla tarczy kołowej (pełnej):

$$\sigma_r(0) = \sigma_\theta(0) \quad \text{lub} \quad u = 0 \quad (18)$$

$$\sigma_r(b) = 0 \quad (19)$$

$$(18) \rightarrow \sigma_r(0) = \sigma_0, \quad C = 0 \quad (20)$$

$$\sigma_r = \sigma_0 - \frac{\gamma}{3g} \omega^2 r^2 \quad (21)$$

$$(19) \rightarrow 0 = \sigma_0 - \frac{\gamma}{3g} \omega^2 b^2 \quad (22)$$

$$\underline{\underline{\omega = \bar{\omega} = \sqrt{\frac{3g\sigma_0}{\gamma b^2}}}} \quad (TG) \quad (23)$$

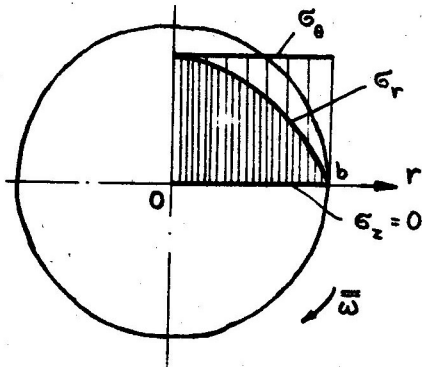
$$HMH \quad \underline{\underline{\bar{\omega} = \sqrt{\frac{3,258 g \sigma_0}{\gamma b^2}}}} \quad (\text{numerycznie}) \quad (24)$$

Sprawdzenie uszeregowania naprężeń:

$$(21), (23) \rightarrow \sigma_r = \sigma_0 - \frac{\gamma}{3g} \omega^2 r^2 = \sigma_0 - \frac{\gamma}{3g} \frac{3g\sigma_0}{\gamma b^2} r^2 \quad (25)$$

$$\sigma_r = \sigma_0 \left(1 - \frac{r^2}{b^2}\right) \quad (26)$$

$$\underline{\underline{0 \leq \sigma_r \leq \sigma_0 = \sigma_0}} \quad \text{zgodnie z założeniem} \quad (27)$$



Warunki brzegowe dla tarczy pierścieniowej swobodnej

$$\sigma_r(a) = 0 \quad (28)$$

$$\sigma_r(b) = 0 \quad (29)$$

$$(16) \rightarrow \sigma_0 - \frac{\gamma}{3g} \omega^2 a^3 + \frac{C}{a} = 0 \quad (30)$$

$$\sigma_0 - \frac{\gamma}{3g} \omega^2 b^3 + \frac{C}{b} = 0 \quad (31)$$

$$C = \frac{\gamma}{3g} \omega^2 a^3 - \sigma_0 a \quad (32)$$

$$(31) \rightarrow \sigma_0 - \frac{\gamma}{3g} \omega^2 b^3 + \frac{\gamma}{3g} \omega^2 \frac{a^3}{b} - \sigma_0 \frac{a}{b} = 0 \quad (33)$$

$$\sigma_0 (b-a) - \frac{\gamma}{3g} \omega^2 (b^3 - a^3) = 0 \quad (34)$$

$$\omega = \bar{\omega} = \sqrt{\frac{3g\sigma_0(b-a)}{\gamma(b^3-a^3)}} \quad (35)$$

$$\bar{\omega} = \sqrt{\frac{3g\sigma_0}{\gamma(b^2+ab+a^2)}} \quad (36)$$

$$(32) \rightarrow C = \frac{\sigma_0 a^3}{b^2+ab+a^2} - \sigma_0 a = - \frac{\sigma_0 ab(a+b)}{b^2+ab+a^2} \quad (37)$$

$$(16) \rightarrow \sigma_r = \sigma_0 \left[ 1 - \frac{r^2}{b^2+ab+a^2} - \frac{ab(a+b)}{b^2+ab+a^2} \frac{1}{r} \right] \quad (38)$$

