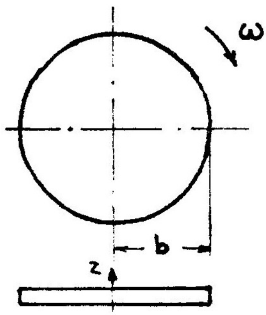
INSTYTUT MECHANIKI
I PODSTAW KONSTRUKCJI MASZYN
WYDZIAŁU MECHANICZNEGO

Wytrzymałość Materiałów III - Wykład 6

Nośność graniczna wirujących tarcz kołowych i pierścieniowych



r. r. w.

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} + \frac{\gamma}{g}\omega^2 r = 0 \qquad (1)$$

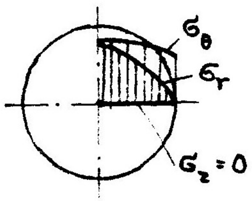
HMH

$$\sigma_r^2 + \sigma_\theta^2 - \sigma_r \sigma_\theta = \sigma_o^2 \qquad (2)$$

$$\text{TG} \quad \sigma_I - \sigma_{III} = \sigma_o \qquad (3)$$

w zakresie sprężystym:

$$\left.\begin{aligned} \sigma_I &= \sigma_\theta \\ \sigma_{II} &= \sigma_r \\ \sigma_{III} &= \sigma_z = 0 \end{aligned}\right\} \qquad (4)$$



zakładamy podobnie:

$$\sigma_I = \sigma_\theta , \qquad \sigma_{III} = 0 .$$

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} + \frac{\gamma}{g}\omega^2 r = 0 \qquad (5)$$

$$\sigma_\theta = \sigma_o = \text{const.} \qquad (6)$$

PK Zam. 1878/73 - 350 egz.

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_0}{r} + \frac{\gamma}{g}\omega^2 r = 0 \tag{7}$$

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r}{r} = \frac{\sigma_0}{r} - \frac{\gamma}{g}\omega^2 r \tag{8}$$

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r}{r} = 0 \tag{9}$$

$$\frac{d\sigma_r}{\sigma_r} = -\frac{dr}{r} \tag{10}$$

$$\ln \sigma_r = -\ln r + \ln \bar{C} \tag{11}$$

$$\sigma_r = \frac{\bar{C}}{r} \qquad\qquad \bar{C} = \bar{C}(r) \tag{12}$$

$$\frac{\bar{C}'}{r} - \frac{\bar{C}}{r^2} + \frac{\bar{C}}{r^2} = \frac{\sigma_0}{r} - \frac{\gamma}{g}\omega^2 r \tag{13}$$

$$\bar{C}' = \sigma_0 - \frac{\gamma}{g}\omega^2 r^2 \tag{14}$$

$$\bar{C} = \sigma_0 r - \frac{\gamma}{g}\omega^2 \frac{r^3}{3} + C \tag{15}$$

$$(12) \rightarrow \quad \sigma_r = \sigma_0 - \frac{\gamma}{3g}\omega^2 r^2 + \frac{C}{r} \tag{16}$$

$$\sigma_\theta = \sigma_0 \tag{17}$$

Warunki brzegowe dla tarczy kołowej (pełnej):

$$\sigma_r(0) = \sigma_\theta(0) \qquad\qquad \text{lub} \quad u = 0 \tag{18}$$

$$\sigma_r(b) = 0 \tag{19}$$

$$(18) \rightarrow \quad \sigma_r(0) = \sigma_0, \qquad C = 0 \tag{20}$$

$$\sigma_r = \sigma_0 - \frac{\gamma}{3g}\omega^2 r^2 \tag{21}$$

$$(19) \rightarrow \quad 0 = \sigma_0 - \frac{\gamma}{3g}\omega^2 b^2 \tag{22}$$

$$\omega = \bar{\bar{\omega}} = \sqrt{\frac{3g\sigma_0}{\gamma b^2}} \qquad \text{(TG)} \tag{23}$$

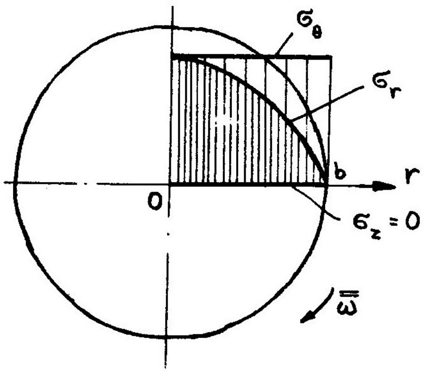
$$\text{HMH} \qquad \bar{\bar{\omega}} = \sqrt{\frac{3{,}258\, g\sigma_0}{\gamma b^2}} \qquad \text{(numerycznie)} \tag{24}$$

Sprawdzenie uszeregowania naprężeń:

$$(21),(23) \rightarrow \sigma_r = \sigma_0 - \frac{\gamma}{3g}\omega^2 r^2 = \sigma_0 - \frac{\gamma}{3g}\,\frac{3g\sigma_0}{\gamma b^2} r^2 \tag{25}$$

$$\sigma_r = \sigma_0\left(1 - \frac{r^2}{b^2}\right) \tag{26}$$

$$0 \leq \sigma_r \leq \sigma_\theta = \sigma_0 \qquad \text{zgodnie z założeniem} \tag{27}$$



Warunki brzegowe dla tarczy pierścieniowej swobodnej

$$\sigma_r(a) = 0 \qquad (28)$$

$$\sigma_r(b) = 0 \qquad (29)$$

(16) →

$$\sigma_0 - \frac{\gamma}{3g}\omega^2 a^2 + \frac{C}{a} = 0 \qquad (30)$$

$$\sigma_0 - \frac{\gamma}{3g}\omega^2 b^2 + \frac{C}{b} = 0 \qquad (31)$$

$$C = \frac{\gamma}{3g}\omega^2 a^3 - \sigma_0 a \qquad (32)$$

(31) →

$$\sigma_0 - \frac{\gamma}{3g}\omega^2 b^2 + \frac{\gamma}{3g}\omega^2 \frac{a^3}{b} - \sigma_0 \frac{a}{b} = 0 \qquad (33)$$

$$\sigma_0 (b-a) - \frac{\gamma}{3g}\omega^2 (b^3 - a^3) = 0 \qquad (34)$$

$$\omega = \bar{\bar{\omega}} = \sqrt{\frac{3g\sigma_0 (b-a)}{\gamma (b^3 - a^3)}} \qquad (35)$$

$$\bar{\bar{\omega}} = \sqrt{\frac{3g\sigma_0}{\gamma (b^2 + ab + a^2)}} \qquad (36)$$

(32) →

$$C = \frac{\sigma_0 a^3}{b^2 + ab + a^2} - \sigma_0 a = -\frac{\sigma_0 ab(a+b)}{b^2 + ab + a^2} \qquad (37)$$

(16) →

$$\sigma_r = \sigma_0 \left[1 - \frac{r^2}{b^2 + ab + a^2} - \frac{ab(a+b)}{b^2 + ab + a^2}\frac{1}{r} \right] \qquad (38)$$