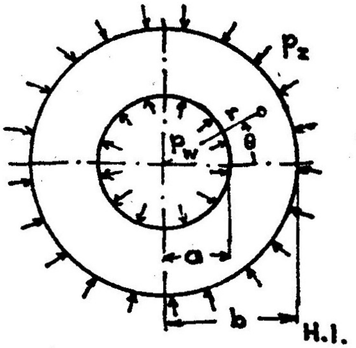
INSTYTUT MECHANIKI
I PODSTAW KONSTRUKCJI MASZYN
WYDZIAŁU MECHANICZNEGO

Wytrzymałość Materiałów III - Wykład 5

Nośność graniczna rur grubościennych pod działaniem ciśnienia



r.r.w.

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \qquad (1)$$

HMH

$$(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_z - \sigma_r)^2 = 2\sigma_0^2 \qquad (2)$$

$$D_\varepsilon = \varphi D_\sigma \quad \rightarrow \quad \varepsilon_r - \varepsilon_\theta = \varphi(\sigma_r - \sigma_\theta) \qquad (3)$$

$$\varepsilon_\theta - \varepsilon_z = \varphi(\sigma_\theta - \sigma_z) \qquad (4)$$

$$\varepsilon_z - \varepsilon_r = \varphi(\sigma_z - \sigma_r) \qquad (5)$$

p.z.o. $\quad \varepsilon_r + \varepsilon_\theta + \varepsilon_z = 0 \qquad (6)$

r. geom.

$$\left.\begin{array}{l} \varepsilon_r = \dfrac{du}{dr} \\ \varepsilon_\theta = \dfrac{u}{r} \end{array}\right\} \rightarrow \varepsilon_r = \frac{d}{dr}(r\varepsilon_\theta) \qquad (7)$$

$$\varepsilon_z = \text{const.} = \varepsilon \qquad (8)$$

$$(6),(7),(8) \rightarrow \quad r\varepsilon_\theta' + \varepsilon_\theta + \varepsilon_\theta + \varepsilon = 0 \qquad (9)$$

$$r\varepsilon_\theta' + 2\varepsilon_\theta = -\varepsilon \qquad (10)$$

$$r\varepsilon_\theta' + 2\varepsilon_\theta = 0 \tag{11}$$

$$\frac{d\varepsilon_\theta}{\varepsilon_\theta} = -2\frac{dr}{r} \tag{12}$$

$$\ln \varepsilon_\theta' = -2\ln r + \ln \bar{C} \tag{13}$$

$$\underline{\varepsilon_\theta = \frac{\bar{C}}{r^2}} \qquad \bar{C} = \bar{C}(r) \tag{14}$$

$$(10) \rightarrow \quad \frac{\bar{C}'}{r} - \frac{2\bar{C}}{r^2} + \frac{2\bar{C}}{r^2} = -\varepsilon \tag{15}$$

$$\bar{C}' = -\varepsilon r \tag{16}$$

$$\bar{C} = -\varepsilon\frac{r^2}{2} + C \tag{17}$$

$$\left.\begin{aligned} (14) &\rightarrow \quad \varepsilon_\theta = \frac{C}{r^2} - \frac{\varepsilon}{2} \\ (7) &\rightarrow \quad \varepsilon_r = -\frac{C}{r^2} - \frac{\varepsilon}{2} \\ (8) &\rightarrow \quad \varepsilon_z = \varepsilon \end{aligned}\right\} \tag{18}$$

$$\left.\begin{aligned} \varepsilon_r - \varepsilon_\theta &= -\frac{2C}{r^2} \\ \varepsilon_\theta - \varepsilon_z &= \frac{C}{r^2} - \frac{3}{2}\varepsilon \\ \varepsilon_z - \varepsilon_r &= \frac{C}{r^2} + \frac{3}{2}\varepsilon \end{aligned}\right\} \tag{19}$$

$$(2), (3), (4), (5) \rightarrow \quad \frac{(\varepsilon_r - \varepsilon_\theta)^2}{\varphi^2} + \frac{(\varepsilon_\theta - \varepsilon_z)^2}{\varphi^2} + \frac{(\varepsilon_z - \varepsilon_r)^2}{\varphi^2} = 2\sigma_0^2 \tag{20}$$

$$\frac{4C^2}{r^4} + \frac{C^2}{r^4} - 3\frac{C\varepsilon}{r^2} + \frac{9}{4}\varepsilon^2 + \frac{C^2}{r^4} + 3\frac{C\varepsilon}{r^2} + \frac{9}{4}\varepsilon^2 = 2\varphi^2\sigma_0^2 \tag{21}$$

$$\frac{6C^2}{r^4} + \frac{9}{2}\varepsilon^2 = 2\varphi^2\sigma_0^2 \tag{22}$$

$$\underline{\underline{\varphi = \varphi(r) = \frac{1}{\sigma_0}\sqrt{\frac{3C^2}{r^4} + \frac{9}{4}\varepsilon^2}}} \tag{23}$$

$$(1) \rightarrow \quad \frac{d\sigma_r}{dr} + \frac{\varepsilon_r - \varepsilon_\theta}{\varphi r} = 0 \qquad (24)$$

$$\sigma_r = -\int \frac{\varepsilon_r - \varepsilon_\theta}{\varphi r} dr = +2C \int \frac{dr}{\varphi r^3} \qquad (25)$$

Warunki brzegowe: $\sigma_r(a) = -p_w$ (26)

$\sigma_r(b) = -p_z$ (27)

$$(26) \rightarrow \quad \sigma_r = -p_w + 2C \int_a^r \frac{d\xi}{\xi^3 \varphi(\xi)} = -p_w + 2C\sigma_0 \int_a^r \frac{d\xi}{\xi^3 \sqrt{\frac{3C^2}{\xi^4} + \frac{9}{4}\varepsilon^2}} \qquad (28)$$

$$\sigma_r = -p_w + 2C\sigma_0 \int_a^r \frac{d\xi}{\xi \sqrt{3C^2 + \frac{9}{4}\varepsilon^2 \xi^4}} \qquad (29)$$

$$\xi^2 = t, \quad 2\xi d\xi = dt \qquad (30)$$

$$\sigma_r = -p_w + C\sigma_0 \int_{a^2}^{r^2} \frac{dt}{t\sqrt{3C^2 + \frac{9}{4}\varepsilon^2 t^2}} \qquad (31)$$

Płaski stan odkształcenia: $\varepsilon_z = \varepsilon = 0$.

$$\sigma_r = -p_w + \frac{C\sigma_0}{\sqrt{3C^2}} \int_{a^2}^{r^2} \frac{dt}{t} = -p_w + \frac{2}{\sqrt{3}}\sigma_0 \ln\frac{r}{a} \qquad (32)$$

gdy $C > 0$, $\sqrt{C^2} = C$

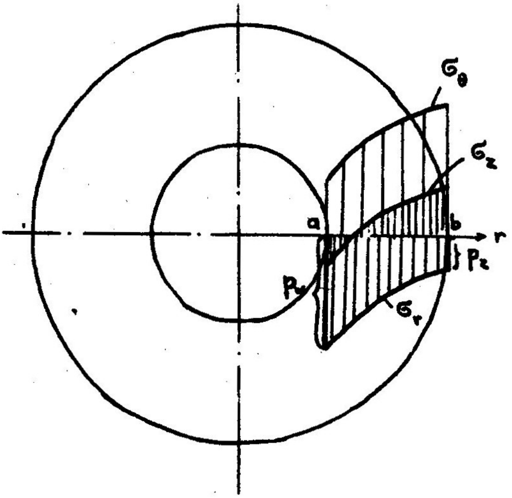
$$(3) \rightarrow \quad \sigma_\theta = \sigma_r - \frac{\varepsilon_r - \varepsilon_\theta}{\varphi} = \sigma_r + \frac{2C}{\varphi r^2} \qquad (33)$$

$$\sigma_\theta = -p_w + \frac{2}{\sqrt{3}}\sigma_0 \ln\frac{r}{a} + \frac{2}{\sqrt{3}}\sigma_0 \qquad (34)$$

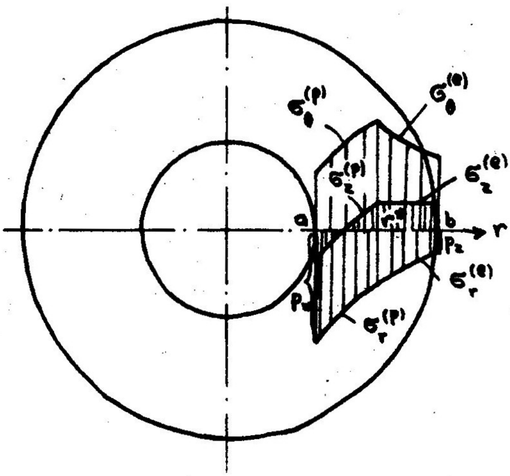
$$(4),(19) \rightarrow \quad \sigma_z = -p_w + \frac{2}{\sqrt{3}}\sigma_0 \ln\frac{r}{a} + \frac{1}{\sqrt{3}}\sigma_0 \qquad (35)$$

$$(27) \rightarrow \quad -p_z = -p_w + \frac{2}{\sqrt{3}}\sigma_0 \ln\frac{b}{a} \tag{36}$$

$$p_w - p_z = \overline{\overline{\Delta p}} = \frac{2}{\sqrt{3}}\sigma_0 \ln\frac{b}{a} \tag{37}$$



Rozkład naprężeń w chwili wyczerpania nośności granicznej



Zakres sprężysto-plastyczny:

$\sigma_r^{(p)}(a) = -p_w$

$\sigma_r^{(e)}(b) = -p_z$

$\sigma_r^{(p)}(r^*) = \sigma_r^{(e)}(r^*)$

$\sigma_i^{(e)}(r^*) = \sigma_0 \quad \rightarrow r^*$

PK Zam. 115/76 - 700 egz.